the lengths of the inspection intervals, e.g., longer inspection intervals in the early life time and shorter inspection intervals at the later service life, so that the maximum benefit can be achieved. The possibility of using or combining various inspection qualities or techniques to achieve either a maximum utility or a maximum improvement of fleet reliability can further be developed. Accordingly, the tradeoff between replacement, inspection quality, inspection interval, inspection frequency, retirement of aircraft, intended service life, etc., presents a broad spectrum of very interesting problems for further study.

V. Conclusion

An optimization scheme for the inspection frequency has been formulated on the basis of the expected-cost-of-failure concept. The optimum inspection frequency is determined by minimizing the expected cost while the constraint on the specified fleet reliability is satisfied. It has been shown that the optimum inspection frequency increases as the relative importance of the cost of inspection compared to the cost of failure becomes smaller, thus increasing the fleet reliability more significantly.

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Comparison of Sonic Boom Minimization Results in Real and Isothermal Atmospheres

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Nomenclature

a = speed of sound
A = equivalent area
CO = characteristic overpressure, $4I/T$
$F(y)$ = Whitham F function
I = impulse, $\int_{p>0} pdt$
L = equivalent length
M = Mach number
p = ambient pressure
Δp = overpressure
P = pressure perturbation
q = dynamic pressure
r = vertical distance from airplane axis
S = ray tube area
t = time, sec
T = total time between front and rear shock waves
W = cruise weight of airplane
x = axial distance
y = axial distance
α = advance
β = $(M^2-1)^{1/2}$
Γ = $(\gamma+1)/2$
ρ = density

Subscripts

h = altitude of initial waveform
g = ground level
L = equivalent length
y = axial distance

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FOR a cruising aircraft in an isothermal atmosphere, Seebass and George have provided a method which minimizes certain features of the pressure signature and yields the corresponding F function and equivalent area distribution, A.[1,2] To provide the same capability for a real atmosphere, their method has been modified using the appropriate equations of George and Plotkin[3] for horizontal advance,

$$\alpha_h = \frac{\Gamma M_h{}^3 F(y)}{(2\beta)^{1/2}} \int_o^r \frac{p_h}{p} \left(\frac{\rho a_h}{\rho_h a}\right)^{1/2} \left(\frac{S_h}{r_h S}\right)^{1/2} \frac{M}{\beta} dr$$

ray tube area,

$$\frac{S_h}{r_h S} = 1/M_h [1 - \frac{1}{M(r)^2}]^{1/2} \int_o^r \frac{dr}{[M(r)^2 - 1]^{1/2}}$$

and signature propagation

$$P_g = \left(\frac{S_h}{S_g}\right)^{1/2} \left(\frac{\rho_g a_g}{\rho_h a_h}\right)^{1/2} P_h$$

These results were programed on a digital computer and numerous calculations have been made for both atmospheres at varying flight conditions, using a scale height of 25,000 ft[1] for the isothermal atmosphere. The results shown in the figures herein are believed to be typical and are limited to pressure signatures in which the maximum overpressure has been minimized for the following conditions: altitude, 60,000 ft (18,288 m); weight, 600,000 lb (272,155 kgm); length, 300 ft (91.44 m); reflection factor, 2. Ratios of overpressure (Δp), impulse (I), and characteristic overpressure[4] *(CO)*, as predicted for the two atmospheres are shown in Fig. 1. For Mach numbers greater than 1.85, the isothermal approximation predicts overpressures given by the real atmosphere to within 1%. Predictions of impulse and characteristic overpressure are less accurate because of differences in signature length as shown for Mach 3 in Fig. 2, but for the same Mach range, these predictions fall within 5% of the real values. At lower supersonic Mach numbers the speed of sound gradient in the real atmosphere causes much more curvature of the ray tube than is predicted by the isothermal atmosphere,[3] thus larger errors occur for isothermal predictions in this Mach number range.

If the effects of aircraft wake and engine exhaust are neglected, and the aircraft volume is zero at its base, the

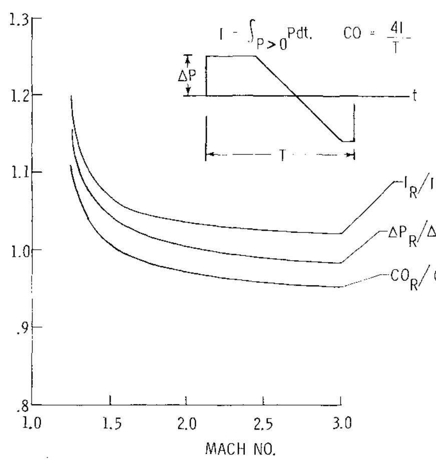

Fig. 1 Comparisons of signature parameters in real and isothermal atmospheres.

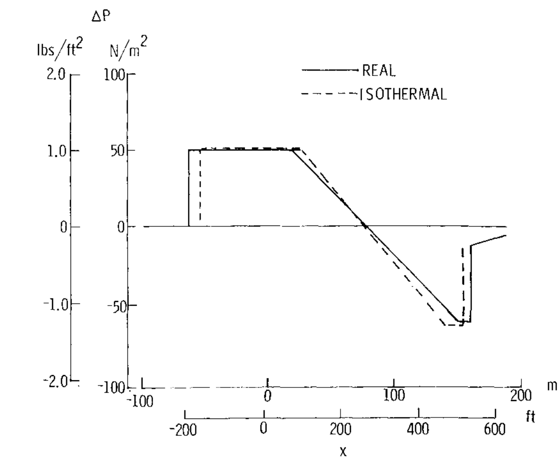

Fig. 2 Signatures resulting from real and isothermal atmospheres.

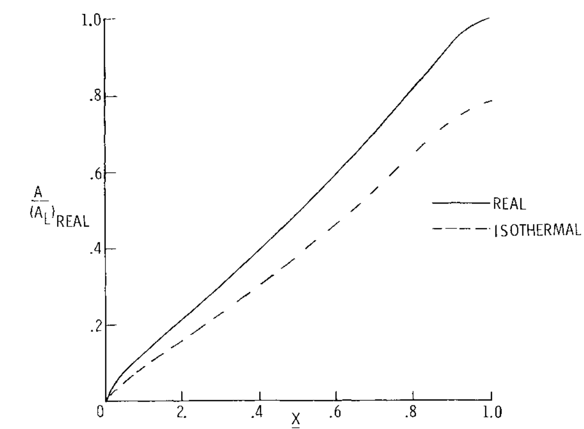

Fig. 3 Normalized area distributions. $A_{L,REAL}$ = 976.263 ft^2 (90.697m^2).

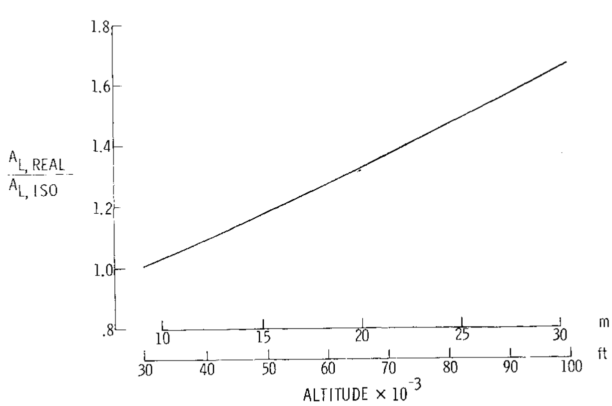

Fig. 4 Correction factor for isothermal area distributions when H = 25,000 ft.

equivalent area, A_L, at the equivalent length, L, is proportional to W/q, where W is the cruise weight and q, the dynamic pressure. At constant Mach number and with a pressure scale height of 25,000 ft, the isothermal approximation for q yields equivalent area distributions that are too low to reflect the cruise weight requirements at the given altitude and Mach number in the real atmosphere (Fig. 3). Within the isothermal approximation, A_L may be corrected in one of two ways: using the correct value for q at the given altitude or selecting the scale height which gives the proper pressure at altitude. For the given conditions at $M=3$, the correct q yields the correct area distribution to within 3% but Δp was overpredicted by 20%. Alternatively, changing the scale height corrects the area distribution to within 1.5% of

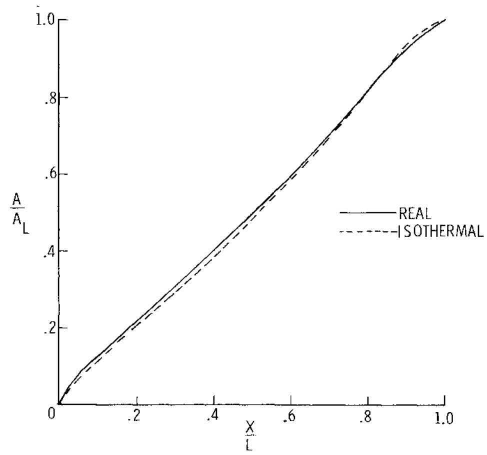

Fig. 5 Comparison of normalized area distributions.

the real distribution but overpredicts Δp by 8%. Thus, it appears that the most accurate prediction of Δp occurs when using a scale height of 25,000 ft in the isothermal atmosphere. The resulting area distribution may be improved by the factor $A_{L,Real}/A_{L,ISO}$. For a scale height of 25,000 ft, values of this ratio as a function of altitude are shown in Fig. 4. After correction, the isothermal distribution differs by less than 8% from the real atmosphere distribution (Fig. 5). This corrected area distribution overpredicts Δp by 5% after propogation through the real atmosphere.

Thus, for sonic boom minimization studies in mid-range supersonic Mach numbers, use of the isothermal atmosphere with a scale height of 25,000 ft provides reliable estimates of overpressures, and a simple adjustment to the isothermal equivalent areas provides a good approximation to the correct area distribution. However, for design studies, propagation of a known F function or minimization studies at low supersonic Mach numbers, the isothermal approximation to the real atmosphere becomes unsatisfactory.

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Static Stability and Aperiodic Divergence

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Nomenclature

B,C,D,E = coefficients of the characteristic equation
g = acceleration due to gravity
I_y = pitch moment of inertia

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